Measuring Rhythm: Which Ruler to Use?

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Background / Objectives

- Often the way to tell the difference between two things is easily understood -for shoes we have size, for places we have distance, and for time we have duration. However, for rhythms there is no clear way to delineate the difference between them.
- Currently, the most commonly used method, beats per minute (BPM), cannot even distinguish between the same rhythm played at two different speeds.
- This poster will examine the concepts of maximally even rhythms, the methods used to measure their evenness and will highlight distinct strengths and weaknesses in each method.
- The mathematical methods developed for the analysis of rhythm comparison and measurement have useful applications to other scientific fields and engineering.

Definitions/ Notation:

Onset - An attack or point of sound!

Offset - A rest or point of silence

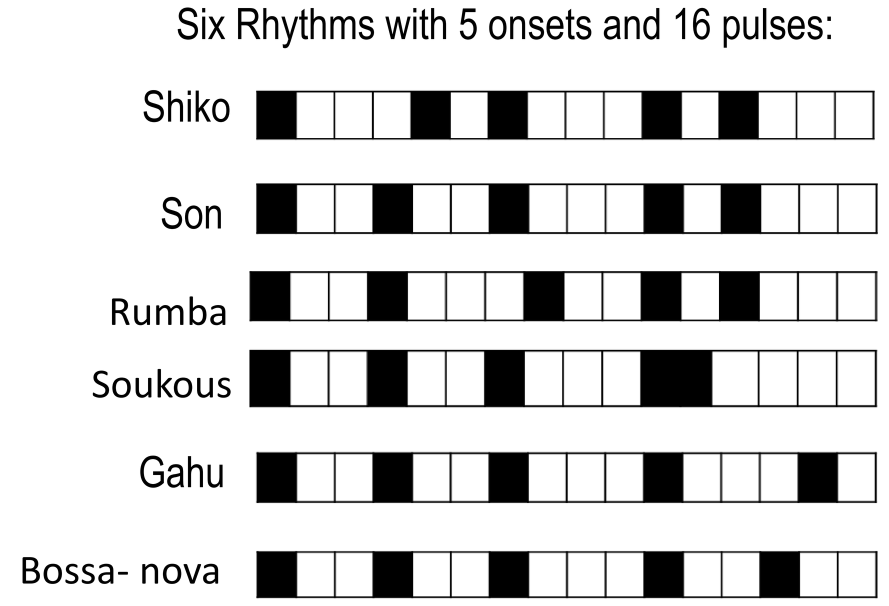
Pulse - A count in a measure

Rhythm – " Rhythm is the perception of both regular and irregular accent patterns and their interactions." – C.B. Manahan and E.C. Carterette

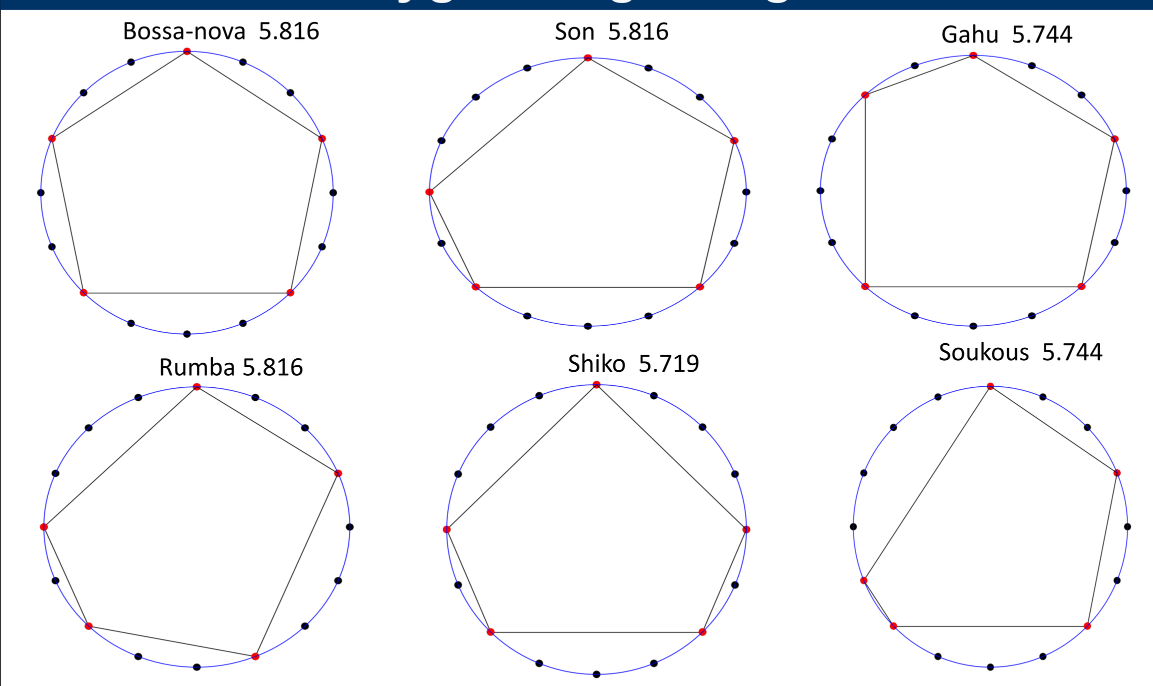
Even Rhythm – A rhythm that is well spaced. Well spaced rhythm is considered to have an equal number of pulses between each onset

Odd Rhythm - not well spaced

Six Rhythms with 5 onsets and 16 pulses:



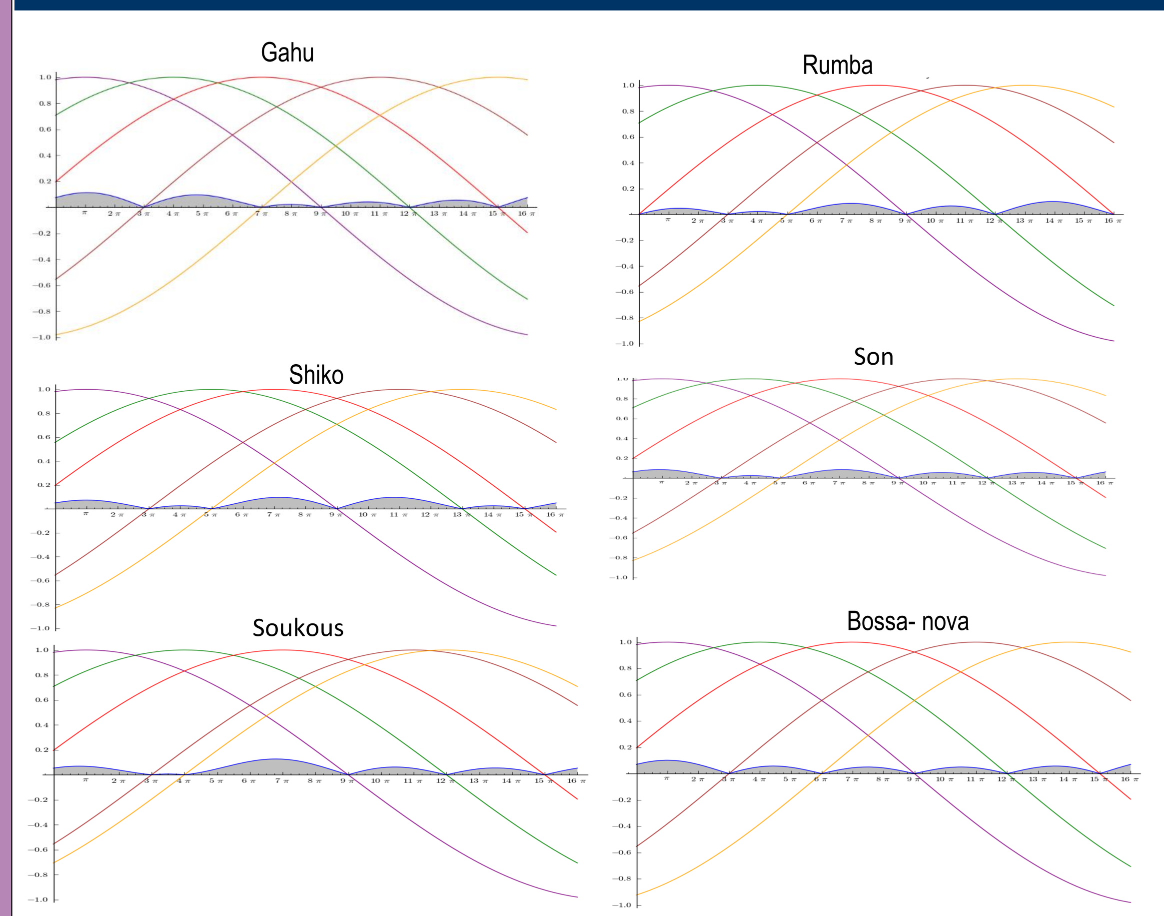
Polygon-edge length:



Description:
•Every rhythm can be measured on a circle due to its periodic behavior
•The 16 points on the circle are pulses and the red points are onsets
•Polygon-edge measures the perimeter of the polygon that is inscribed inside the circle of radius 1.

Interesting Characteristics
•Maintains the same value when rhythm is played forward or backward
•Invariant under rotational symmetry
•May produce the same value for differing rhythms
•Does NOT produce a clear order for the six rhythms (From most even to least even the order is as follows)
1. Bossa-nova 2. Son-Rumba-Gahu 3. Shiko 4. Soukous

Cosine Wave:



Description:

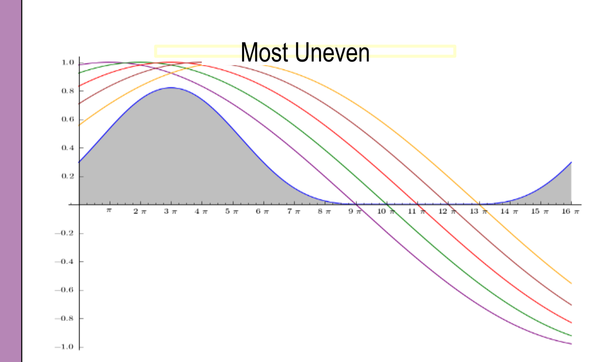
- The cosine wave method represents each rhythm as a product of five cosine functions (one for each onset) with period 16π (for 16 pulse rhythms).
- Each individual cosine function is adjusted so that the peak of the curve corresponds to the timing of the onset.
- Taking the product of these functions leads to creative and destructive interactions between individual cosine functions.
- Finally, to generate a number, the integral of the absolute value of the product is taken from 0 to 16π

Cosine Wave Continued:

Interesting Characteristics:

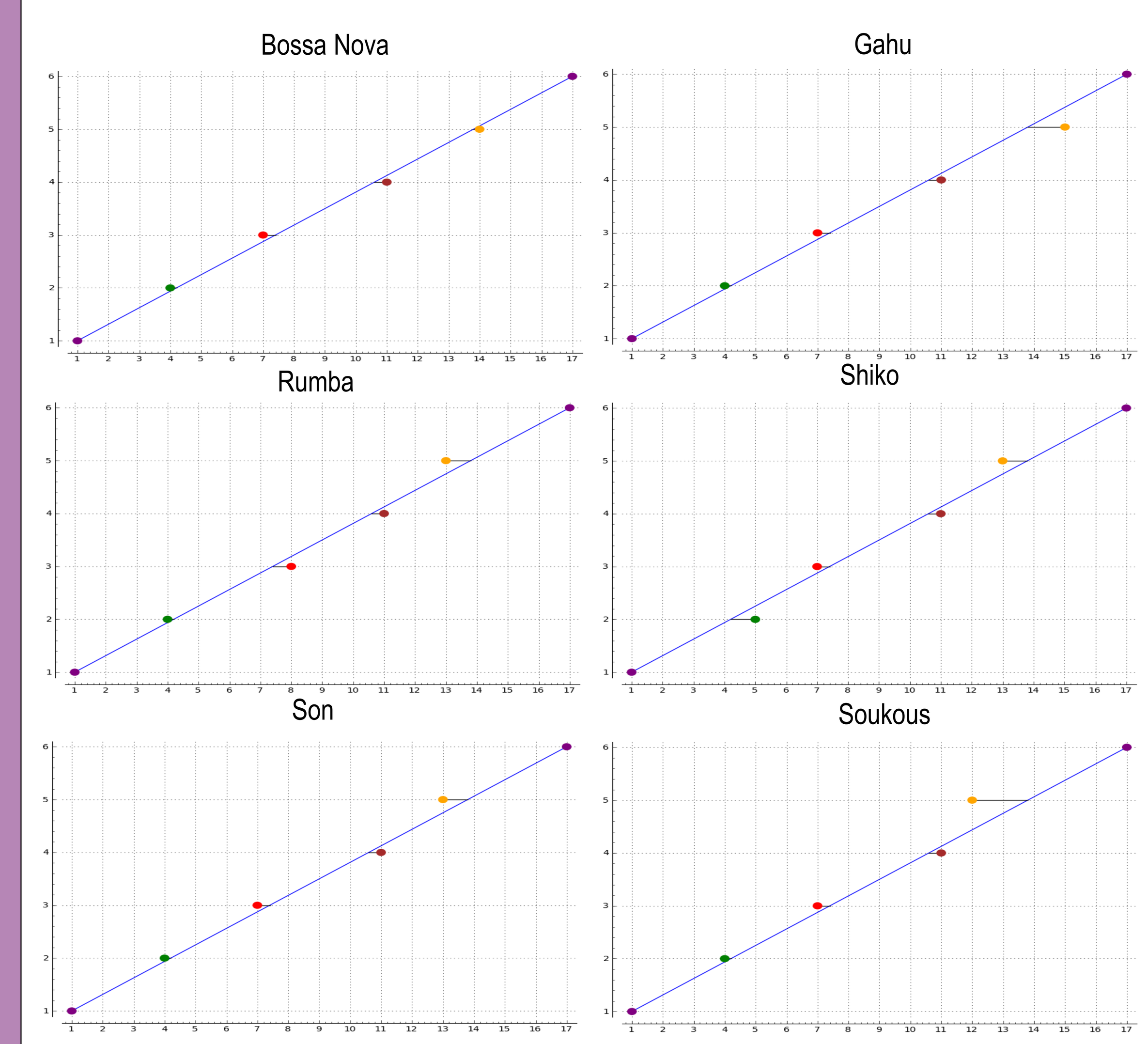
- This method also provides the same value for the rhythms when they are played forward and backward
- Invariant under rotational symmetry
- Produces an ordering of the six rhythms (From most even to least even) :

1. Bossa-nova 2. Son 3. Rumba 4. Shiko 5. Gahu 6. Soukous



The figure to the left represents the most uneven (5, 16) rhythm under this measurement (This rhythm has 5 onsets and 16 pulses)

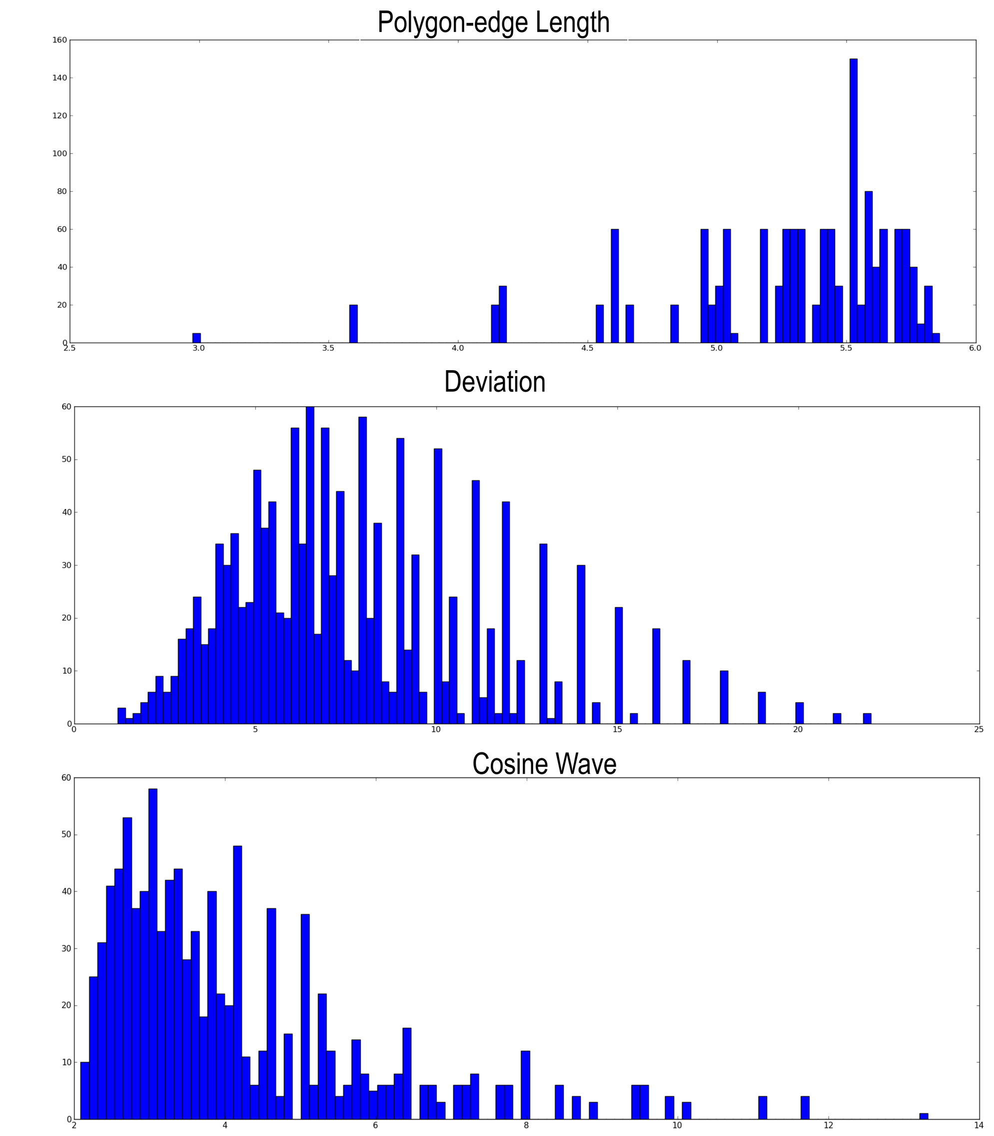
Deviation Method:



Description:
•The deviation method sums the distance from each onset to a line representing the sixteen-beat period divided into five equal length pieces.
•In this way, if all five onsets occur at equal intervals, they would all appear on the line.
• This method sums the horizontal distances from each onset to the ideal line.

Interesting Characteristics:
•Method provides same value for rhythms played forward and backward
•It is NOT invariant under rotational symmetry
•Produces a clear ordering of the six rhythms (From most even to least even):
1. Bossa-nova 2. Son 3. Rumba 4. Gahu 5. Shiko 6. Soukous

Histograms:



Algorithm

- The data for the histograms is produced by an algorithm which cycles through all possible 5 onset 16 pulse rhythms finding the value associated with the given measurement method for each.
- It is clear that the algorithm will generate at least one iteration of each rhythm.
- It is not clear how many times the algorithm repeats rhythms that should be considered the same under rotation or reflection.
- The principal aim of generating the histograms is to exam how each method distributes the values of the rhythms.

Histogram Characteristics

- The deviation method clearly has an advantage over the other two in that it spreads the data out farther suggesting clearer distinctions between rhythms when using this measure.
- The deviation method may not do as good of a job at distinguishing between different rhythms as the cosine wave method, which has less spread, but less repetition of values.

Further Work:

1. Count all 16 pulse rhythms with 5 onsets, when you consider two rhythms the same if they are the same under periodic or reflective symmetry.
2. Write an efficient algorithm to cycle through these rhythms.
3. Expand to more complicated representations of rhythm. How would these measures adapt or change when two sounds were used to form the rhythm? What about three? etc.

References

G. T. Toussaint, The Geometry of Musical Rhythm, Chapman and Hall/CRC, January 2013.

Godfried Toussaint, "The Geometry of Musical Rhythm," Proceedings of the Japan Conference on Discrete and Computational Geometry, (JCDCG 2004), LNCS 3742, Springer-Verlag, Berlin-Heidelberg, 2005, pp. 198-212.